Metamagnets in uniform and random fields

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Abstract

We study a two-sublattice Ising metamagnet with nearest and next-nearest-neighbor interactions, in both uniform and random fields. Using a mean-field approximation, we show that the qualitative features of the phase diagrams are significantly dependent on the distribution of the random fields. In particular, for a Gaussian distribution of random fields, the behavior of the model is qualitatively similar to a dilute Ising metamagnet in a uniform field.

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I. INTRODUCTION

The random-field Ising model has been a considerable source of research over the last twenty years[1−3]. Systems with quenched random fields are experimentally realized in antiferromagnets with bond mixing or site dilution[4,5]. A large variety of these systems have been subjected to detailed experimental studies[6].

Although most theoretical problems associated with the ferromagnetic Ising model in a random field (as the lower critical dimension, the pinning effects, and the existence of a static phase transition) have been solved, some questions are still open. In particular, there is still room to investigate the existence of a tricritical point[7−9] and the exact relation to the dilute antiferromagnet in a uniform field. Depending on the choice of the random-field distribution, the mean-field approximation gives rise to a tricritical point (which is present for a symmetric double-delta distribution[10], but does not occur in the case of a Gaussian form[11]). On the basis of the central limit theorem, some hand-wave arguments can be used to support the physical relevance of the Gaussian distribution (the tricritical point produced by the double-delta functions being a mere artifact of the mean-field approximation).

The proof of the equivalence between an Ising ferromagnet in a random field and a dilute antiferromagnet in a uniform field is based on renormalization-group arguments that can be applied for weak fields[4,5]. In the mean-field approximation[12](or in the equivalent and exactly soluble model with infinite-range interactions[13,14]), it is possible to establish a complete mapping between the parameters of the Ising ferromagnet in a random field and the dilute Ising antiferromagnet or metamagnet in a uniform field. In particular, it is known that the random fields should be associated with a symmetric double-delta distribution for arbitrary dilution[12−14], including the pure case where there is no dilution! This peculiar result suggests that, instead of describing the random fields generated by dilution, the mean-field approximation is just referring to the two-sublattice structure of the antiferromagnet (which is reflected in the symmetric double-delta distribution of the random fields). It should be mentioned that the mean-field approximation for the dilute Ising metamagnet in a uniform

field suffers from other difficulties when confronted with Monte Carlo calculations[15–17] and experimental results[18,19]. Whereas numerical simulations and experiments indicate that the first-order transition is destroyed when the dilution is increased, no such effect is predicted in a mean-field calculation.

In this paper we use a mean-field approximation to consider an Ising metamagnet with nearest and next-nearest-neighbor interactions, in a uniform field and a random field. This model is equivalent to a dilute Ising metamagnet in a field for an appropriate choice of the random field distribution. Since the exact mapping of the dilution to the random fields is unknown, only a qualitative comparison can be made. We consider double-delta and Gaussian random-field distributions. The behavior of the model and the phase diagrams depend very much on these random-field distributions. The Gaussian form seems to be more appropriate for a description of the diluted system.

II. DEFINITION OF THE MODEL

We consider a regular lattice of N sites, with Ising spins $S_i = \pm 1$ at each site, that can be divided into two equivalent interpenetrating sublattices, A and B. The z nearest neighbor (nn) spins of a given spin are on the other sublattice, while the z' next-nearest neighbor (nnn) spins are all on the same sublattice. The Hamiltonian of the system is given by

$$\mathcal{H} = J \sum_{\text{nn}} S_i S_j - J' \sum_{\text{nnn}} S_i S_j - \sum_i (H + H_i) S_i, \tag{2.1}$$

where J is the nn exchange parameter, the sum $\sum_{\text{nn}}$ is over all pairs of nn spins, J' is the nnn exchange parameter, the sum $\sum_{\text{nnn}}$ is over all nnn spins, H is the strength of the external uniform magnetic field, and H_i is the strength of the local random field. We assume that the nn interactions are antiferromagnetic ($J > 0$), the nnn interactions are ferromagnetic ($J' \geq 0$), and the local random fields H_i are uncorrelated. Even though it is possible to consider sublattice-dependent probability distributions, in this paper we use the same probability distribution at every site.

III. MEAN-FIELD EQUATIONS

We derive the mean-field equations from Bogoliubov's variational principle[20],

$$\langle F\rangle_{\rm av} \le \langle F_{\rm t}\rangle_{\rm av} + \langle\langle \mathcal{H}-\mathcal{H}_{\rm t}\rangle_{\rm t}\rangle_{\rm av}, \tag{3.1}$$

where $\langle\cdots\rangle_{\rm av}$ denotes averaging over the random-field distribution and $\langle\cdots\rangle_{\rm t}$ the thermal averaging with respect to the trial Hamiltonian $\mathcal{H}_{\rm t}$. Choosing the non-interacting trial Hamiltonian

$$\mathcal{H}_{\rm t} = -\sum_i (H+H_i)S_i - \eta_{\rm A}\sum_{i\in{\rm A}} S_i - \eta_{\rm B}\sum_{i\in{\rm B}} S_i, \tag{3.2}$$

where $\eta_{\rm A}$ and $\eta_{\rm B}$ are the variational parameters, we obtain

$$\begin{aligned}\langle F\rangle_{\rm av} \le &-\frac{N}{2\beta}\langle \ln 2\cosh\beta(H+H_i+\eta_{\rm A})\rangle_{\rm av}\\ &-\frac{N}{2\beta}\langle \ln 2\cosh\beta(H+H_i+\eta_{\rm B})\rangle_{\rm av} + \frac{JNz}{2}m_{\rm A}m_{\rm B}\\ &-\frac{J'Nz'}{4}(m_{\rm A}^2+m_{\rm B}^2) + \frac{N}{2}\eta_{\rm A}m_{\rm A} + \frac{N}{2}\eta_{\rm B}m_{\rm B},\end{aligned} \tag{3.3}$$

with

$$m_{\rm A} = \langle \tanh\beta(H+H_i+\eta_{\rm A})\rangle_{\rm av}, \tag{3.4a}$$

$$m_{\rm B} = \langle \tanh\beta(H+H_i+\eta_{\rm B})\rangle_{\rm av}. \tag{3.4b}$$

The condition that the right-hand side of Eq. (3.3) is stationary determines the variational parameters,

$$\eta_{\rm A} = -Jzm_{\rm B} + J'z'm_{\rm A}, \tag{3.5a}$$

$$\eta_{\rm B} = -Jzm_{\rm A} + J'z'm_{\rm B}. \tag{3.5b}$$

Inserting Eqs. (3.5a)-(3.5b) into Eqs. (3.4a)-(3.4b), we arrive at the mean-field equations,

$$m_{\rm A} = \langle \tanh\beta(H+H_i-Jzm_{\rm B}+J'z'm_{\rm A})\rangle_{\rm av}, \tag{3.6a}$$

$$m_{\rm B} = \langle \tanh\beta(H+H_i-Jzm_{\rm A}+J'z'm_{\rm B})\rangle_{\rm av}. \tag{3.6b}$$

The right-hand side of Eq. (3.3) at the stationary point gives the mean-field free energy per spin,

$$\begin{aligned} f = &-\frac{1}{2\beta}\langle \ln 2\cosh\beta(H + H_i - Jzm_{\rm B} + J'z'm_{\rm A})\rangle_{\rm av} \\ &-\frac{1}{2\beta}\langle \ln 2\cosh\beta(H + H_i - Jzm_{\rm A} + J'z'm_{\rm B})\rangle_{\rm av} \\ &-\frac{Jz}{2}m_{\rm A}m_{\rm B} + \frac{J'z'}{4}(m_{\rm A}^2 + m_{\rm B}^2). \end{aligned} \tag{3.7}$$

IV. LANDAU EXPANSION

In this Section we develop the Landau expansion along the same steps used for the pure case[21]. It is convenient to introduce the reduced quantities

$$t = \frac{1}{\beta(Jz + J'z')}, \quad h = \frac{H}{Jz + J'z'}, \quad h_i = \frac{H_i}{Jz + J'z'}, \tag{4.1}$$

and the parameters

$$\epsilon = \frac{J'z'}{Jz} \geq 0, \quad \gamma = \frac{-Jz + J'z'}{Jz + J'z'} = \frac{\epsilon - 1}{\epsilon + 1}. \tag{4.2}$$

In terms of the uniform and staggered magnetizations,

$$M = \frac{m_{\rm A} + m_{\rm B}}{2}, \quad m_{\rm s} = \frac{m_{\rm A} - m_{\rm B}}{2}, \tag{4.3}$$

the mean-field equations (3.6a) and (3.6b) can be written as

$$M = \frac{1}{2}\left[\langle \tanh\frac{1}{t}(h + h_i + \gamma M + m_{\rm s})\rangle_{\rm av} + \langle \tanh\frac{1}{t}(h + h_i + \gamma M - m_{\rm s})\rangle_{\rm av}\right], \tag{4.4}$$

and

$$m_{\rm s} = \frac{1}{2}\left[\langle \tanh\frac{1}{t}(h + h_i + \gamma M + m_{\rm s})\rangle_{\rm av} - \langle \tanh\frac{1}{t}(h + h_i + \gamma M - m_{\rm s})\rangle_{\rm av}\right]. \tag{4.5}$$

Also, the free energy per spin, given by Eq. (3.7), may be written in the form

$$\begin{aligned} f = &-\frac{t}{2}\langle \ln 2\cosh\frac{1}{t}(h + h_i + \gamma M + m_{\rm s})\rangle_{\rm av} \\ &-\frac{t}{2}\langle \ln 2\cosh\frac{1}{t}(h + h_i + \gamma M - m_{\rm s})\rangle_{\rm av} - \frac{\gamma}{2}M^2 + \frac{1}{2}m_{\rm s}^2. \end{aligned} \tag{4.6}$$

Let us now write the uniform magnetization as $M = M_0 + m$, where M_0 is the paramagnetic solution, given by equation

$$M_0 = \langle \tanh \frac{1}{t}(h + h_i + \gamma M_0) \rangle_{\rm av}. \tag{4.7}$$

The expansion of the right-hand side of Eqs. (4.4) and (4.5) in powers of $(\gamma m \pm m_{\rm s})$ gives the expressions

$$m = \frac{1}{2} \sum_{n=1}^{\infty} A_n[(\gamma m + m_{\rm s})^n + (\gamma m - m_{\rm s})^n], \tag{4.8a}$$
$$m_{\rm s} = \frac{1}{2} \sum_{n=1}^{\infty} A_n[(\gamma m + m_{\rm s})^n - (\gamma m - m_{\rm s})^n], \tag{4.8b}$$

where the coefficients A_n are given by

$$A_1 = -\frac{1}{t}(T_2 - 1), \tag{4.9a}$$
$$A_2 = \frac{1}{t^2}(T_3 - T_1), \tag{4.9b}$$
$$A_3 = -\frac{1}{3t^3}(3T_4 - 4T_2 + 1), \tag{4.9c}$$
$$A_4 = \frac{1}{3t^4}(3T_5 - 5T_3 + 2T_1), \tag{4.9d}$$
$$A_5 = -\frac{1}{15t^5}(15T_6 - 30T_4 + 17T_2 - 2). \tag{4.9e}$$

with

$$T_k = \langle \tanh^k \frac{1}{t}(h + h_i + \gamma M_0) \rangle_{\rm av}. \tag{4.10}$$

We now determine m in terms of $m_{\rm s}$ in the form

$$m = B_1 m_{\rm s}^2 + B_2 m_{\rm s}^4 + B_3 m_{\rm s}^6 + \dots. \tag{4.11}$$

Inserting this expansion into Eq. (4.8a), and equating the coefficients of same degree in $m_{\rm s}$, we find the coefficients B_n in terms of A_n. Finally, substituting m, given by Eq. (4.11), into Eq.(4.8b) we obtain the expansion

$$a m_{\rm s} + b m_{\rm s}^3 + c m_{\rm s}^5 + \cdots = 0, \tag{4.12}$$

where

$$a = 1 - A_1, \tag{4.13a}$$

$$b = \frac{2\gamma A_2^2}{\gamma A_1 - 1} - A_3, \tag{4.13b}$$

$$c = \frac{2\gamma^3 A_2^4}{(\gamma A_1 - 1)^3} - \frac{9\gamma^2 A_2^2 A_3}{(\gamma A_1 - 1)^2} + \frac{6\gamma A_2 A_4}{\gamma A_1 - 1} - A_5. \tag{4.13c}$$

The second order transition is found at $a = 0$ with $b > 0$. The tricritical point occurs for $a = b = 0$ with $c > 0$.

In the absence of random fields the model exhibits a tricritical point[21] in the $h - t$ phase diagram for $\epsilon > 3/5$. In the numerical calculations of the next sections, we just consider the case $\epsilon = 1$, which is typical for the range of values $\epsilon > 3/5$.

V. PHASE DIAGRAMS FOR THE DOUBLE-DELTA DISTRIBUTION

In this Section we study the phase diagrams for the case of a double-delta distribution,

$$P(h_i) = \frac{1}{2}[\delta(h_i - \sigma) + \delta(h_i + \sigma)]. \tag{5.1}$$

Fig. 1 shows the phase diagrams in the $h - t$ plane for various values of the randomness σ.

The case of no randomness ($\sigma = 0$) corresponds to the pure Ising metamagnet[21]. The phase diagram comprises a metamagnetic phase ($m_{\rm s} \neq 0$) at low fields and a paramagnetic phase ($m_{\rm s} = 0$) at high fields. The transitions between these phases are first-order for low temperatures and second-order for high temperatures, being separated by a tricritical point[21] at $t = 2/3$, as shown in Fig. 1(a).

For $\sigma > (2/3)\tanh^{-1} 1/\sqrt{3} = 0.438\ldots$, there is a second tricritical point at lower fields, as illustrated by Figs. 1(d)–(e). Also, for the randomness in the interval $0 < \sigma < 0.5$, there is a first-order transition line inside the metamagnetic phase at low temperatures. Through this transition line the staggered magnetization decreases discontinuously as the field is increased. This internal first-order transition line ends at a critical point. Finally, for $\sigma > 0.5$, the internal and lower first-order transition lines merge into a single first-order transition line ending at the tricritical point, as illustrated in Fig. 1(f).

For the particular case of the double-delta distribution and $\epsilon = 1$ or $\gamma = 0$ that we are considering, the phase diagrams in the $\sigma - t$ plane are exactly the same as in the $h - t$ plane. This comes from the invariance of Eqs. (4.5) and (4.6), for the staggered magnetization and the free energy, respectively, under the interchange between h and σ (and from the independence of the free energy on the uniform magnetization M). Therefore, Fig. 1 also represents the phase diagrams in the $\sigma - t$ plane if we interchange h and σ throughout this figure and in its caption. The phase diagram comprises a metamagnetic phase ($m_\mathrm{s} \neq 0$) for small σ and a paramagnetic phase ($m_\mathrm{s} = 0$) for high values of σ. In particular, the phase diagram for $h = 0$ is equivalent (after flipping all the spins on a sublattice) to the diagram of a ferromagnetic Ising model in a double-delta random field[10].

VI. PHASE DIAGRAMS FOR THE GAUSSIAN DISTRIBUTION

Now we study the phase diagrams for the Gaussian distribution,

$$P(h_i) = \frac{1}{\sqrt{2\pi}\sigma} \exp(-\frac{h_i^2}{2\sigma^2}). \tag{6.1}$$

In Fig. 2, we show the $h - t$ plane for various values of the randomness σ. Again, the case of no randomness ($\sigma = 0$) corresponds to the pure Ising metamagnet[21], with a first-order separated from a second-order transition line by a tricritical point at $t = 2/3$. The tricritical temperature decreases as the randomness is increased until $\sigma = 0.5$, when the transition between the metamagnetic and paramagnetic phases becomes everywhere of second-order. The similarity of these phase diagrams as a function of σ with those of a dilute metamagnet as a function of dilution[15–17] is quite striking. It suggests that a Gaussian random field gives, at least qualitatively, a good description of the random fields generated by dilution in a metamagnet.

In Fig. 3, we show the phase diagram in the $\sigma - t$ plane for various values of the uniform field h. The case $h = 0$ is equivalent (after flipping all the spins on a sublattice) to the ferromagnetic Ising model in a Gaussian random field[11]. The transition line is of second-order for all temperatures and it crosses the σ axis at $\sigma = \sqrt{2/\pi} = 0.79\ldots$. For large σ the

transition at low fields becomes first-order and a tricritical point separates the first-order and the second-order lines. For still larger randomness, the transition becomes first-order always.

VII. CONCLUSIONS

We have used the mean-field approximation to show that the phase diagrams of an Ising metamagnet in the presence of a uniform and of random fields are strongly dependent on the form of the distribution of probabilities of the random fields. In particular, if the model exhibits a first-order transition in zero random field, then a double-delta distribution never destroys this first-order transition, in contradistinction to the case of a Gaussian distribution. In this respect, there is a striking similarity in the qualitative behavior of the metamagnet in a Gaussian random field and a dilute metamagnet. This suggests that, by keeping the two-sublattice structure and choosing an appropriate random field distribution, we can give a better description of the dilute metamagnet than the previous mean-field studies that map dilute Ising metamagnets in a uniform field into Ising ferromagnets in a double-delta distribution of random fields.

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FIGURES

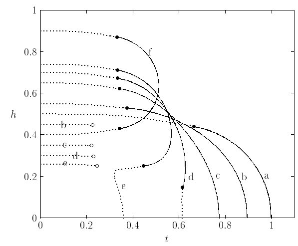

FIG. 1. Phase diagrams in $h-t$ plane in the case of a double-delta distribution for (a) $\sigma = 0$, (b) $\sigma = 0.3$, (c) $\sigma = 0.4$, (d) $\sigma = 0.45$, (e) $\sigma = 0.49$ and (f) $\sigma = 0.65$. The solid lines represent continuous transitions. The dashed lines are first-order transitions. The filled circles are tricritical points, and the empty circles are critical points.

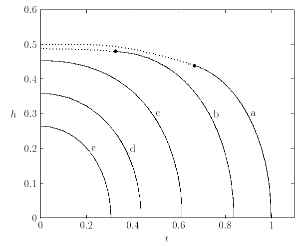

FIG. 2. Phase diagrams in $h-t$ plane in the case of a Gaussian distribution for (a) $\sigma = 0$, (b) $\sigma = 0.4$, (c) $\sigma = 0.6$, (d) $\sigma = 0.7$ and (e) $\sigma = 0.75$. The solid lines represent continuous transitions. The dashed lines are first-order transitions. The filled circles are tricritical points.

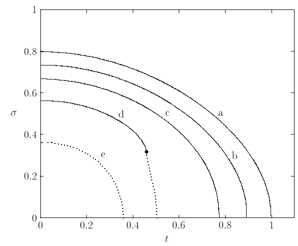

FIG. 3. Phase diagrams in $\sigma - t$ plane in the case of a Gaussian distribution for (a) $h = 0$, (b) $h = 0.3$, (c) $h = 0.4$, (d) $h = 0.47$ and (e) $h = 0.49$. The solid lines represent continuous transitions. The dashed lines are first-order transitions. The filled circle is a tricritical point.